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THE FUNDAMENTAL LAW OF THE GRATING.

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BY

JANET TUCKER HOWELL.

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The Fundamental Law of the Grating.

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The Coincidence Method.

The fundamental law of the grating stated by J. von Fraunhofer in 1823 is that $N\lambda = w(\sin\theta + \sin\varphi)$ where N is the order of the spectrum, λ is the wavelength, w is the distance apart of the lines of the grating from centre to centre, θ is the angle between the incident light and the normal to the grating, and φ is the angle between the diffracted light and the normal. H. A. Rowland[1] in his paper on the theory of concave gratings shows that, if the grating and the illuminated slit both lie on a circle, the diameter of which is equal to the radius of curvature of the grating the spectra of different orders will all be in focus on the same circle. If the photographic plate is placed on this circle directly opposite to the grating $\varphi = 0$ and the law of gratings is that $N\lambda = w \sin\theta$. It follows,therefore, that, for any one angle θ, $N\lambda$ = constant. A line in the first order will then coincide with one of half the wavelength in the second, and one of one third the wavelength in the third order, and so on. It is therefore possible, by photographing superimposed spectra, to measure the wavelength of lines of any order if certain standard wavelengths are known in one order from which the scale of the

~~scale of the~~ plate, in millimeters per A. U., may be determined for that order. This method of measuring the wavelengths of lines in superimposed spectra of different orders is called the coincidence method. It was the method used by Rowland in determining his table of standards and in parts of the spectrum in which standards have not been established by independent interferometer measurements it is still the only method for the determination of wavelengths.

Other Work on Grating Measurements.

H. Kayser.

When Rowland's table of standards first appeared, claiming an accuracy of .005 A.U. it was thought that his values would never suffer any serious modifications. But when A. A. Michelson determined the wavelengths of the three cadmium lines and C. Fabry and A. Perot, using the red cadmium line as the primary standard, determined a number of secondary standards in the iron spectrum by an interferometer method, it was found that their values did not agree with Rowland's. The difference was of the order of .19 A.U. and varied considerably instead of being proportional to the wavelength. Kayser, in his first paper on Standards of Wavelength[2], took up the question of the errors in Rowland's Table of Standards. They arise he says from two sources: First, Rowland was convinced that the displacement observed in going from solar

to arc spectra was due to the accidental movement of the apparatus so he shifted different parts of the spectrum to smaller or greater wavelengths to make the two spectra agree. He then determined other parts from these by the method of coincidences so that errors may have accumulated to a different extent in different parts of the spectrum. If this were the only source of error it would be of little importance for then new determinations made by the method of coincidences using the interferometer lines as standards would be correct within the .005 A.U. claimed by Rowland plus the errors of the standard lines. Michelson[3] had, however, shown that an error in the ruling of the grating would vitiate the results of the coincidence method, so Kayser thought that errors in Rowland's tables might have been due also to a second cause; namely, the unreliability of the method of coincidences, and the object of his work was to test this method. Kayser possesses two of Rowland's largest concave gratings, both with a radius of 21 feet and a width of 5.5 inches, one of them ruled with 16,000 lines to the inch, the other with 20,000. With both gratings he photographed the region containing the Fabry and Perot standards from λ 5302 to λ5434 in the second order and determined from them the superimposed third order lines from λ3550 to λ3630. He then photographed λ3550 to λ3630 in the fourth order and determined from them the Fabry and Perot

lines at λ4736 and λ 4859 in the third order. He found that for one grating the difference between his values and those of Fabry and Perot amounted to .019 A.U. while for the other it amounted to .108 A.U. If the Fabry and Perot standards were correct these results showed that neither of his gratings were available for the method of coincidences. He then compared the two gratings directly. He selected the region between the secondary standards λ 5302 and λ 5434 in the second order and determined the wavelengths of the third order lines in that region. He found that the values obtained with the two gratings differed by a large but approximately constant amount the mean of which was .033 A.U. He concluded therefore that neither of his gratings was available for coincidence work and he thought it highly improbable that gratings existed perfect enough for accurate measurements by this method.

This large error found by Kayser in comparing his two gratings has never been satisfactorily explained. One source of error in wavelength measurement lies in the possibility of selecting poor lines. If the lines are double with one component weaker than the other, or if they belong to the type that is displaced and unsymmetrically widened under pressure, there will be large errors introduced by the uncertain estimation of the centre of gravity. But in this case the difference between the values found for the two gratings was of the same order of magnitude for all the lines measured, so

that the error must have arisen from some other cause. Kayser attributed the difference to the fact that both gratings were bad, which is of course possible. Michelson[3] has published a paper discussing the spectra of imperfect gratings. Changes in the strength of the ruled lines or in their spacing would affect the distribution of light in the spectral lines which are images of a finite width. When the spacing is exact the spectral images are symmetrical. If the spacing is not exact but the strength of the rulings is constant the spectral images will in general be unsymmetrical. It is possible by a sudden release of strain in some part of the course to have all subsequent rulings slightly displaced with reference to the first part. There will then be a constant error introduced in comparing one order of spectrum with another, the actual error depending on the actual linear displacement of the rulings, the order of spectrum, and the number of lines in the grating. An error in the grating spacing of one of Kayser's gratings would then account for the fact that the third order lines as calculated from the second order standards with one grating differ by a constant quantity from the same lines calculated with the other grating. The error found in comparing the two gratings directly may therefore be accounted for by an error in the grating spacing of one of them. But even the better of the two

gratings gave a difference of .019 A.U. between the values of the standards obtained by the coincidence method and the values of Fabry and Perot. And as from the grating work done up to that time there seemed no other way of accounting for this error Kayser came to the conclusion that both of his gratings were too imperfect for coincidence work and confined his wavelength measurements to direct interpolation between Fabry and Perot standards of the same order. In his paper published in 1910[4], he gives the values of a large number of tertiary standards calculated by direct interpolation between secondary standards. He adopted as secondary standards the mean of the measurements of Fabry and Buisson, Eversheim and Pfund, and each line was interpolated between as many pairs of standards as possible and the mean was taken. His list of tertiary standards extends from λ 4118 to λ 6495.

F. Papenfus.

Since Kayser's last paper work has been done on this subject in several directions. Papenfus in 1911[5] published two papers on the availability of the coincidence method for wavelength measurement. He worked with a Rowland concave grating having a 13 foot radius and 10,000 lines to the inch. He first determined the values of the tertiary standards from λ6065 to λ6678 by direct interpolation and com-

pared his results with Kayser's. For the most part he finds that their results differ by less than .01 A.U., which is the greatest error that would be permissible. This difference he attributed to the fact that they used different methods of measuring their plates. Kayser interpolated each line between as many pairs of standards as possible and took the mean, while Papenfus used three standards for each interpolation, the middle one giving a correction factor to be applied to the value obtained from the other two. Then, having satisfied himself that his values for the tertiary standards were on the whole in good agreement with Kayser's he investigated the error introduced by using the coincidence method instead of direct interpolation. Using his tertiary standards in the second order to establish a correction curve between the secondary standards of the second order, he measured the superimposed lines of the third order in this region by the coincidence method. The lines thus determined ran from λ4076 to λ4376 a region which Kayser had also covered in his measurements by the method of interpolation. Comparing his values with Kayser's he found that the differences were of the same order of magnitude as the differences between their tertiary standards, being for the most part less than .01 A.U. He concludes therefore that there was no error introduced by the coincidence method itself in the case of his grating and advises all owners of concave gratings to examine their gratings in this manner so that a more general proof of

the method may be obtained.

F. Goos.

Goos has done some recent wavelength work with a plane grating having 7000 lines to the inch[6]. His work was undertaken because he considered it important to have the tertiary standards determined by as many different observers and in as many ways as possible. He investigated the region from λ 4282 to λ 5324 and on comparing his values with Kayser's he finds a systematic error which he attributed to the fact that he uses later values for the standards[7] and measures his plates in a different way. The method of measurement he uses is as follows: Adopting the dispersion formula $\lambda = a + bx + cx^2$ where λ is the wavelength and x the reading of the screw, he determines the constants a b and c for each plate from measurements on three standard lines. Then using these constants he calculates the values of the other standards and by comparing them with the international values obtains a correction curve for his plate. Having corrected for the systematic difference between his values and Kayser's he finds that the remaining differences are small, the average variation being ± .0035 A.U. In his second paper he investigates the effect of varying the light source. In his first work he used a short arc, about 3 or 4 mm. and by means of a screen cut out all except the middle part. When with the same current he used about 2 mm. of the centre of a long arc (8 - 9 mm.) he found that the character of

many of the lines changed. With the shorter arc the lines were broader and when the broadening was unsymmetrical the values found with the two types of arc differed, sometimes by as much as .02 A.U. Similar line displacements he says may be expected if different parts of the arc are used or if the current or voltage is changed, or if iron rods of different sizes are used. So he arrives at the conclusion that a variation in the light source used will account for large variations in the values found for the tertiary standards, and that before these tertiary standards can be established with any degree of accuracy a more exact definition of the standard arc is necessary, a definition which will be difficult because in different parts of the spectrum different portions of the arc are better suited to the work.

C. E. St. John and L. W. Ware.

The most recent work on tertiary standards has been done at Mt. Wilson by St. John and Ware[8]. They worked with a plane grating and, using the secondary standards of the international system, determined the values of the tertiary standards from λ 5371 to λ 6494. They worked both at Pasadena and Mt. Wilson and found that, owing to the difference ~~owing to height~~ in pressure amounting to about one-fifth of an atmosphere, due to the height of Mt. Wilson, some of the lines were appreciably displaced and unsymmetrically widened. The

mean difference for the lines suffering great displacement towards the red was .048 A.U. per atmosphere. Others were displaced towards the violet with a mean displacement of the same orders of magnitude. The secondary standards were displaced only .006 A.U. per atmosphere and in the region investigated they found 25 lines which were symmetrically widened and only slightly displaced under pressure. These they recommended as tertiary standards. Comparing the values for these 25 lines found by Kayser, Papenfus, Goos and St. John, we find that only in the case of four lines is the maximum difference from the mean as great as .01 A.U. The mean maximum difference is .005 A.U. We may conclude then that all the lines available as tertiary standards between λ 5371 and λ 6494 are known accurately to hundredths of an Ångström Unit.

Summary.

The work on iron standards up to this time has brought considerable evidence against but also some evidence in favor of the adaptability of the coincidence method to wavelength measurement. Kayser found a systematic error amounting to .033 A.U. between third order lines determined by the coincidence method from second order standards, with two separate gratings. This could be accounted for by supposing one of the gratings to have suffered a linear displacement of the rulings of one part with respect to the rest, were it not for the fact

that in calculating the Fabry and Perot standards by the coincidence method he found an error amounting to .019 A.U. even for his better grating. He therefore abandoned all idea of calculation by the method of coincidences and confined his work to direct interpolation between standards.

This work of Goos on the variation of wavelength with the length of arc and that of St. John and Ware on the displacement and unsymmetrical broadening of some lines under pressure, shows that there is a chance of large errors in wavelength measurements if the experimental conditions vary to any extent. It leads one to suspect that after all the inadaptability of one or both of Kayser's gratings to coincidence work may not have been altogether responsible for the errors found. In calculating the Fabry and Perot standards by the coincidence method it is possible, that, although the standards themselves might not be subject to any appreciable variation in wavelength, the intermediate tertiary standards used might belong to the type that is subject to displacement and unsymmetrical broadening under pressure. As these intermediate tertiary standards lay between λ3550 and λ3630, a region which has not been investigated in this respect, there is yet no conclusive evidence on this point.

There is one source of error in coincidence work for which correction should be made in any absolute determinations of wavelength. This error is due to the fact that the dispersion of air varies with

temperature. The following examples will show the magnitude of the effect: $n = \frac{\lambda_e}{\lambda}$ where n is the refractive index, λ_e is the wavelength in aether, and λ is the wavelength in air. Suppose that the wavelength $\lambda 5640$ first order and λ 2820 second order coincide exactly at 0° and 760 mm. pressure.

Then $\lambda_{e_1} = n \times 5640 = 1.0002924 \times 5640$

and $\lambda_{e_2} = n \times 2820 = 1.0003091 \times 2820$

At 10° and 760 mm. the values for n are different

$$\lambda_1 = \frac{\lambda_{e_1}}{n} = \frac{1.0002924 \times 5640}{1.0002761} = 5640.09187$$

$$\lambda_2 = \frac{\lambda_{e_2}}{n} = \frac{1.0003091 \times 2820}{1.0002919} = 2820.04847$$

$$\therefore 2\lambda_2 - \lambda_1 = 5640.0969 - 5640.0919 = .005 \text{ A.U.}$$

So that two lines which coincide exactly at 0° will differ by .005 A.U. at 10°. This shows the importance of keeping the temperature constant and may account to a large extent for the variations found in measurements by the coincidence method. In exact determinations of wavelength by this method all values should be reduced to standard conditions.

Papenfus, in comparing first his wavelength measurements by direct interpolation and then the measurements by the coincidence method with Kayser's values determined by direct interpolation, found the same order of difference in both cases, which shows that his grating is adapted to coincidence work. It is possible that one of Kayser's gratings

may also be adapted to this work but the coincidence method is far from being reinstated in the position of importance it enjoyed at the time of Rowland's work. It was with the purpose of further testing the method that the piece of work here reported was undertaken.

Results of this Investigation:

The work consists of two parts. In the first part, using the international standards from λ 5266 to λ 5434 in the second order I determined from them by the method of coincidences the superimposed third order lines in this region with two gratings, one having 15,000 the other 20,000 lines to an inch. This is the same region that Kayser used in comparing his 20,000 and 16,000 line gratings. My results agreed remarkably well and show that the two gratings examined are perfect enough for wavelength determinations by the method of coincidences. But this part of the work did not in itself constitute a thorough proof of the law of gratings. In the second part of my paper I have shown that it is possible starting with one standard line and assuming the law of gratings to determine the value of secondary standard lines to such a degree of accuracy that it would be feasible, if the secondary standards were not already determined by independent methods, to start from one known line and build up a whole system of wavelengths from it, provided of course that the gratings used in the

work were as good as the ones with which I have been working. This second part of my work shows that a system of wavelengths can be built up from one line by means of the coincidence method which will attain the degree of accuracy originally claimed for this method by Rowland and therefore verifies the fundamental law of gratings.

Apparatus:

After the death of Rowland his two ruling machines were not used until 1910, when they were again put into working order by Dr. J. A. Anderson. Several of the gratings ruled last year by Dr. Anderson have been at my disposal. I have had therefore exceptional opportunities for carrying on grating work. I only regret that I have not had the time to test a larger number of them in the same and in different directions. In the first part of my work, comparing tertiary standards obtained by the coincidence method with two gratings, I used one of Rowland's six inch gratings with a 21 foot radius and 20,000 lines to the inch, and one of Anderson's six inch gratings with approximately the same radius and 15,000 lines to the inch. In the second part of my work it was necessary to work with superimposed spectra of higher orders, so I used one of Anderson's gratings having a radius of 21 feet and 7,500 lines to the inch.

The grating mounting was that used by Rowland and Jewell

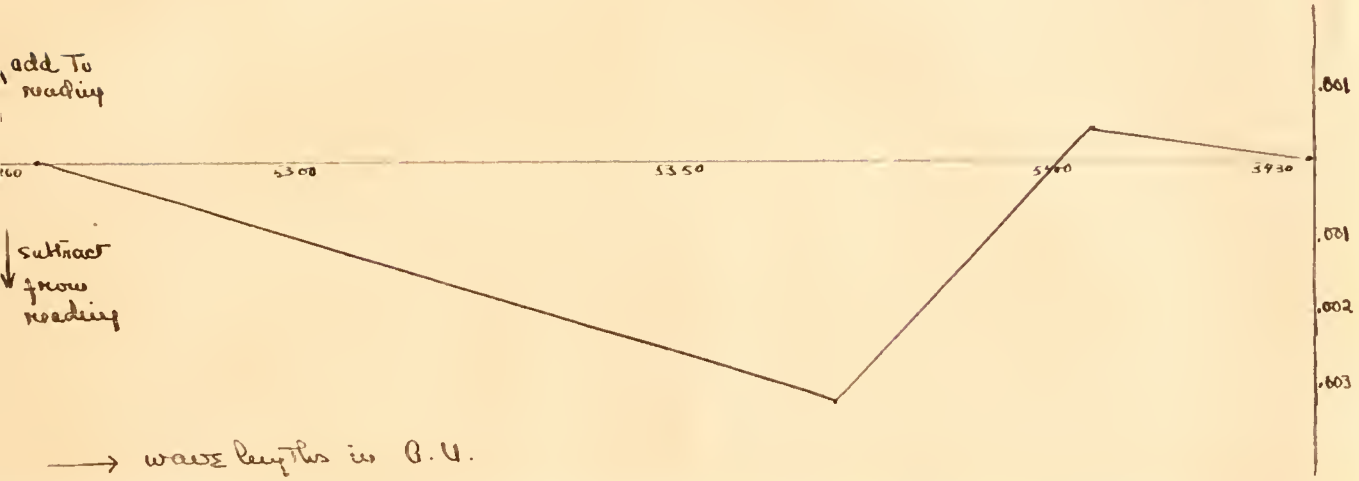
in their work. The experimental conditions were kept as constant as possible so that any line displacement due to a variation in the light source would occur to the same extent throughout the work. In the event therefore of a standardization of the light source to be employed in wavelength measurements my values could not be used for tertiary standards. They are, however, correct in relation to each other and this is all that is necessary in comparing the values obtained from different gratings in a proof of the coincidence method. The iron arc used was about 4 mm. long and was operated at 110 volts with a current between 5.5 and 6.5 amperes.

Owing to the fact that the Johns Hopkins University is situated in the middle of a city it was found impossible to use plates taken in the daytime as the vibrations due to the traffic seriously impaired the sharpness of the lines. All the plates used for measurement were therefore taken between twelve and six A.M. The method of measuring the plates was as follows: The secondary standards λ 5266.569 and λ 5434.527 were used to determine the scale of each plate in the second order. Using the scale thus determined the values of the intermediate standard lines were calculated. From the differences between the calculated and standard values a calibration curve of each plate was drawn from which the readings for all other lines were corrected. A typical calibration curve is shown below in Fig. I.

Fig. II

Typical Calibration Curve
for a plate between
λ 5266 and λ5434

ord. = mm.
abs. = wave lengths
in A.U.



The measuring engine was specially constructed for use in this piece of work. The screw was ground by the Rowland method until all periodic errors were sufficiently eliminated for the purpose, and it was estimated that the errors outside of the error of the screw did not amount to more than 1/5th of a division on the screw head. One division on the screw head equals .001 mm. and, as it was found impossible to count on successive readings agreeing to -

within more than .005 mm., the error of the instrument is negligible in comparison with the error of setting on a line. The readings were made to thousandths of a millimeter and from twelve to fifteen readings were made on each line. The plates were reversed after the whole region had been measured in one direction three times.

The scale of my plates was such that for the 20,000 line grating 1 mm. = .9702 A.U. and for the 15,000 line grating 1 mm = 1.30729 A. U. in the second order. After correcting the readings for each line from the calibration curve of the plate the wavelengths of the lines were found in terms of this scale in the second order and then reduced to third order values by multiplying by the factor $\frac{2}{3}$.

First Part of the Work. Results.

The region between λ 5266 and λ 5434 was measured as described above. In this region there is only one second order line which is on the list recommended by St. John and Ware as tertiary standards. It was therefore the only second order line I measured and my values are given below in Table I together with those obtained by other observers.

Table I.

20,000 G.	15,000 G.	Kayser	Goos	St. John
5429.7005	5429.7001	5429.701	5429.700	5429.702

As there was no way of telling which of the third order lines were the best for this work, and the least affected by varying experimental conditions, I measured all those which were sharp on both sets of plates. The reading for any one line on one plate did not differ from the mean value of the four plates used by more than .003 mm. The greatest difference found between the values of any third line calculated by the two gratings amounted to .005 A.U., a difference which occurs only once in the 47 lines measured. The sum of the differences amounts to + .0620 and - .0237 A.U. The average difference being only $+ \frac{.0383}{47} = +.0008$ A.U. and the average variation $= \pm \frac{.0857}{47} = \pm .0018$ A.U. So the difference between two lines measured on different gratings by the coincidence method is of the same order of magnitude as the difference between values of the same line measured on different plates. This proves that the two gratings worked with were perfect enough to be used in wavelength measurement by the coincidence method. The tabulated results are given below in Table II.

Table II.

20,000 grating	15,000 grating	Difference	Kayser	Rowland
3513.822	3513.8226	+.0006	3513.821	3513.97
3521.2669	3521.2674	+.0005	3521.26	3521.41
3524.0803	3524.0809	+.0006	3524.07	3524.22
3524.2463	3524.2469	+.0006	3524.24	3524.39
3526.0470	3526.0453	-.0017	3526.03	3526.18
3526.1705	3526.1721	+.0016	3526.23	3526.38
3526.2843	3526.3839	-.0004	3526.38	3526.53
3526.6785	3526.6806	+.0021	3526.66	3526.81
3527.7399	3527.8033	+.0034	3527.78	3527.93
3529.8235	3529.8265	+.0030	3529.80	3529.95
3533.0115	3533.0103	-.0012	3533.00	3533.15
3533.2028	3533.2057	+.0029	3533.19	3533.34
3536.5597	3536.5632	+.0035	3536.55	3536.70
3540.1314	3540.1349	+.0035	3540.12	3540.27
3541.0925	3541.0939	+.0014	3541.09	3541.24
3542.0819	3542.0787	-.0032	3542.08	3542.23
3545.6437	3545.6487	+.005	3545.62	3545.77
3553.7459	3553.7432	-.0027	3553.72	3553.87
3554.125	3554.1238	-.0012	3554.11	3554.26
3554.9299	3554.9265	-.0034	3554.92	3555.07
3556.8822	3556.8831	+.0009	3556.880	3557.03
3558.5213	3558.5211	-.0002	3558.52	3558.67
3565.3862	3565.388	+.0018	3565.38	3565.53
3570.1049	3570.1045	-.0004	3570.12	3570.27
3571.9985	3571.9968	-.0017	3572.00	3572.15
3575.3745	3575.3750	+.0005	3575.35	3575.50
3581.2017	3581.2024	+.0007	3581.20	3581.35
3582.2069	3582.2061	-.0008	3582.19	3582.34
3584.6657	3584.6672	+.0015	3584.65	3584.80
3584.9628	3584.9636	+.0008	3584.96	3585.11
3585.3255	3585.3274	+.0019	3585.32	3585.47
3585.7118	3585.7142	+.0024	3585.71	3585.86
3586.1172	3586.1165	-.0007	3586.11	3586.26
3586.9921	3586.9918	-.0003	3586.97	3587.12
3589.1115	3589.1103	-.0012	3589.10	3589.25
3594.6419	3594.6397	-.0022	3594.62	3594.77
3603.2095	3603.2106	+.0011	3603.20	3603.35
3605.4595	3605.4615	+.0020	3605.47	3605.62
3606.6846	3606.6865	+.0019	3606.682	3606.83
3608.8645	3608.8621	-.0024	3608.85	3609.01
3610.1584	3610.1597	+.0013	3610.16	3610.31
3612.0805	3612.0851	+.0046	3612.08	3612.23

Table II (continued)

20,000 grating	15,000 grating.	Difference	Kayser	Rowland
3617.7899	3617.7939	+.0040	3617.78	3617.93
3618.3899	3618.3923	+.0024	3618.38	3618.53
3618.7723	3618.7730	+.0007	3618.77	3618.92
3621.4634	3621.4655	+.0021	3621.46	3621.61
3622.0061	3622.0088	+.0027	3622.00	3622.15

Second Part. (Theory).

In this part of my work I have tried to furnish a real proof of the accuracy with which the fundamental law of the grating holds. The grating used in this part of the work was one of Anderson's with 7,500 lines to the inch and a radius of 21 feet. For my fundamental line I chose λ 5232.957, one of the international secondary standards and proceeded as follows to determine the scale of my plate, knowing only this one wavelength and the law of gratings. I first photographed this line in the 5th order. Let us call the plates of this type No. 1.

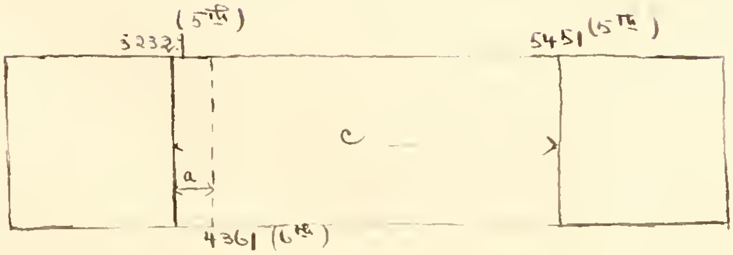


Plate I.

On plate I we know the wavelength of line 5232.957 in the 5th order. Near it is a line in the 6th order which we will call 4361, although its wavelength is not known. The distance between 5232.957 (5th) and 4361 (6th) is "a" mm. Then, if x mm. = 1 A. U. in the 5th order, the distance $a = \frac{a}{x}$ A.U.

The wavelength of the line 4361 will then be $= \left(5232.957 + \frac{a}{x}\right)$ A.U. in terms of the 5th order, which

$$= \left(5232.957 + \frac{a}{x}\right) \frac{5}{6} \text{ A. U. in the 6th order.}$$

The grating is now shifted so as to bring the line 4361 in the 5th order on the middle of the plate. Plates of this type we will call No. II.

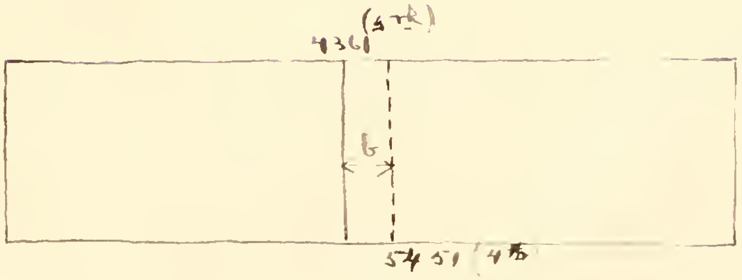


Plate II.

At a distance b mm from 4361 in the 5th order there will be a line in the 4th order which we will call 5451. This line also occurs on Plate I in the 5th order at a distance c mm. from 5232.957 the standard line.

$$b = \frac{b}{x} \text{ A. U. in the 5th order.}$$

$$\therefore 5451 = \left[5232.957 + \frac{a}{x}\right]\frac{5}{6} + \frac{b}{x} \quad \text{A. U. in the 5th order.}$$

$$= \left(\left[5232.957 + \frac{a}{x}\right] \frac{5}{6} + \frac{b}{x} \right) \frac{5}{4} \quad \text{A. U. in the 4th order.}$$

Also from Plate I

$$5451 = \left(5232.957 + \frac{c}{x} \right) \text{ A. U. in the 5th order.}$$

$$\therefore \left(5232.957 + \frac{c}{x} \right) = \left(\left[5232.957 + \frac{a}{x}\right] \frac{5}{6} + \frac{b}{x} \right) \frac{5}{4}$$

$$\therefore 5232.957\, x = 24\, c - 25\, a - 30\, b$$

$$\therefore x = \text{no. of mm. which} = 1 \text{ A. U. in the 5th order.}$$

$$= \frac{24\, c - 25\, a - 30\, b}{5232.957}$$

In paactise I used four lines in the neighborhood of 5232.9 instead of the one referred to above as 4361. Then for the lines given as 5451 I used one of the international standards λ 5455.614 so that my scale, obtained entirely by the coincidence method, might be more easily compared with the scale gotten directly from the two

standards λ 5232.957 and λ 5455.614. In order to identify the lines it was of course necessary to take plates with color screens to cut cut all except certain orders. I then took four plates of type I and four of type II. The values for *b* for each of the four lines used in place of 4361 were then averaged for the four plates of type II, each line being read nine times. These average values for *b* were then used with the different values for *a* and *c* obtained from each of the four plates of type I. The values for *a* and *c* and the average values of *b* are given below in Table III. The four lines used for 4361 are denoted by the symbols I, II, III, IV. On two plates there were only three lines good enough for accurate measurement.

Table III.

	I	II	III	IV	
Plate $I_{(a)}$					
a	-9.2226	+7.7708	+17.3393	+26.0642	
b	+11.2161	-2.9545	-10.9239	-18.1977	
c					212.5546
Plate $I_{(b)}$					
a	-9.2228	+7.769	+17.342	+26.0616	
b	+11.2161	-2.9545	-10.9239	-18.1977	
c					212.5507
Plate $I_{(c)}$					
a		+7.7761	+17.3419	+26.067	
b		-2.9545	-10.9239	-18.1977	
c					212.550
Plate $I_{(d)}$					
a	-9.2215		+17.3412	+26.066	
b	+11.2161		-10.9239	-18.1977	
c					212.5534

The values for x, the number of mm. which equal 1 A. U. in the fifth order are given in Table IV. The Subscripts I,II,III,IV indicate which of the four lines was used in obtaining x, and the mean value of x obtained by the method of coincidences is then compared with x (standard), the value found by assuming the two international secondary standards λ 5232.957 and λ 5455.614 in the usual way.

Table IV.

Plate	$X_{(1)}$	$X_{(II)}$	$X_{(III)}$	$X_{(IV)}$	X_{mean}	$X_{standard}$	Difference
$I_{(a)}$	.954607	.954656	.954631	.954649	.954636	.954628	-.000008
$I_{(b)}$	.95459	.954614	.95460	.954614	.954604	.954610	+.000006
$I_{(c)}$		.954598	.95461	.954614	.954607	.954610	+.000003
$I_{(d)}$	.954595		.954617	.954618	.95461	.95462	+.00001

The greatest error for the scale of the plates determined by this method amounts to one part in a hundred thousand, and the mean error to three parts in a million. It is, however, easier to judge the method by comparing the values for the standard line 5455.614 as found by the coincidence method with the accepted value as found by independent interferometer measurements. This comparison is made below in Table V.

Table V.

Plate	C (mm.)	$\frac{c}{x_{mean}} = C$ (A.U.)	5232.957 +C(A.U.) = st. line calc.	Standard	Difference
$I_{(a)}$	212.5546	222.6553	5455.6123	5455.614	-.0017
$I_{(b)}$	212.5507	222.6585	5455.6155	5455.614	+.0015
$I_{(c)}$	212.5500	222.65707	5455.61407	5455.614	+.00007
$I_{(d)}$	212.5534	222.6599	5455.6169	5455.614	+.0029

We therefore see that the maximum error introduced by calculating one standard line from another by the method of coincidences is .0029 A.U. and the mean error amounts to only .0007 A.U.

In calculating the line 5455.6 from the original line 5232.9 there has of course been no calibration correction made. But even in starting with a set of the International Standards it is necessary to use two of them without calibration correction of any kind in order to get the scale of the plate. In order to carry out the scheme of building up a whole system of wavelengths from one line it would be necessary, after establishing a second line in the manner described above, to determine the intermediate standard lines by coincidences in the same way starting from each of the two fundamental standards as base line and also from each new standard as it is determined. In this way, by averaging the results, each line will be established with

sufficient accuracy to be used as a standard line and from these standards a calibration curve may be drawn for the plate. Then the other lines of all the superimposed spectra in the region can be determined. These lines will give starting points for similar work in other parts of the spectrum. Of course as the steps involved in the work carry one further and further from the two original lines errors might accumulate to a very appreciable extent, but at the same time every new line that is established will give another reference line from which the next lines can be determined. So that by tying up each new line by coincidences with many other lines, some of greater some of less wavelength, the errors introduced will offset each other.

My original intention in taking up this work was to build up, in some part at least, a system such as this. To build up an entire system, determining each line with a sufficient number of coincidences to ensure the desired accuracy, would be a stupendous task. It would also be an unnecessary one because the values obtained for the standard lines could not obtain an accuracy greater than that of the International system. It is therefore simpler to use these standards to begin with. I have shown, however, that, having these standards throughout part of the spectrum, it is possible to investigate the region into which they do not extend by means of the coincidence

method with an accuracy as great as has been obtained as yet in wave-length determination by direct interpolation. Before a grating can be used for this work it must be tested by the method given in Part II of this paper in order to see how closely it obeys the law of gratings. The results obtained with my three gratings have been so good that it is perhaps not too much to say that a grating which is unfit for coincidence work is the exception and not the rule.

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Life.

Janet Tucker Howell, the daughter of William Henry and Janet Tucker Howell, was born in Baltimore, Maryland, on January the first 1889. She received her preparatory school education at the Bryn Mawr School in Baltimore where she was a student from September 1895 until June 1906. In the autumn of 1906 she entered Bryn Mawr College and was the holder of the Bryn Mawr School Scholarship for two years. In June 1910 she received the degree of Bachelor of Arts from Bryn Mawr College. From October 1910 until June 1913 she has been a graduate student in Physics, Astronomy, and Mathematics at the Johns Hopkins University, and in June 1912 she received the degree of Master of Arts.